Stability Analysis of Integer-order Interval System using Kharitonov Theorem

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Abstract: This paper provides a detailed analysis of stability analysis of integer order interval system using Kharitonov theorem. Detailed formulation of khartionov theorem has been discussed. An example of integer order interval system has been provided and khartionov system has been applied to it.

Keywords: Integer order system, stability analysis, Khartionov theorem

Introduction

Kharitonov theorem is one of the most widely used theorem to access robust stability of an integer order interval system. Interval system, its theory and application has been detailed in [1]. The application of interval analysis to robust control has been discussed in [2-3]. Let us consider an integer order interval system represented as

$$G_{HOIS}(s)=\frac{\sum_{i=0}^{n-1}\left[n_i^-,n_i^+\right]s^i}{\sum_{i=0}^{n}\left[d_i^-,d_i^+\right]s^i} \tag{1}$$

Eq(1) can be represented as

$$G(s,P,Q)=\frac{N(s,P)}{D(s,Q)}=\frac{\left[P_0^-,P_0^+\right]+\left[P_1^-,P_1^+\right]s+..+\left[P_{n-1}^-,P_{n-1}^+\right]s^{n-1}}{\left[Q_0^-,Q_0^+\right]+\left[Q_1^-,Q_1^+\right]s+...+\left[Q_n^-,Q_n^+\right]s^n} \tag{2}$$

Asymptotic stability has been discussed in [4] whereas generalized Kharitonov theorem has been discussed in [5]. Necessary condition and complete derivation of Khartionov theorem has been discussed in [6-8, 10, 11].

This paper provides the analysis of Kharitonov theorem with zero inclusion principle. The kharitonov stability of different interval transfer function has been proved using different examples.

KhartionovTheorem

Routh-Hurwitz criterion is an easy method to check the stability of a single polynomial. Kharitonov theorem can be called as an extension of Routh stability criterion to interval polynomials. The Kharitonov theorem states that in an interval polynomial family, which has an infinite number of members, is Hurwitz stable; if and only if a finite small set of four polynomials known as Kharitonov polynomials of the family are Hurwitz stable.

Let us assume a polynomial

$$p(s) = p_o + p_1 s + p_2 s^2 + ... + p_{n-1} s^{n-1} + p_n s^n \quad (3)$$

The parameters of polynomial vary within minimum and maximum as X and Y respectively, i.e. $p_i \in [X_i, Y_i]\ i = 0,1,2,...,n$

The four kharitonov polynomial can be formed as

$$\begin{aligned} K_1(s) &= X_o + X_1 s + Y_2 s^2 + Y_3 s^3 + ... \\ K_2(s) &= X_o + Y_1 s + Y_2 s^2 + X_3 s^3 + ... \\ K_3(s) &= Y_o + X_1 s + X_2 s^2 + Y_3 s^3 + ... \\ K_4(s) &= Y_o + Y_1 s + X_2 s^2 + X_3 s^3 + ... \end{aligned} \quad (4)$$

The polynomial is divided in terms of even and odd polynomial which can be represented as

$$p(s) = p_e(s) + p_o(s) \quad (5)$$

The even and odd polynomials are represented in s-domain as

$$\begin{aligned} p_e(s) &= p_o + p_2 s^2 + p_4 s^4 + ... \\ p_o(s) &= p_1 + p_3 s^3 + p_5 s^5 + ... \end{aligned} \quad (6)$$

The even and odd polynomials are represented in frequency domain as

$$\begin{aligned} p_e(\omega) &= p_o - p_2 \omega^2 + p_4 \omega^4 \\ p_o(\omega) &= p_1 - p_3 \omega^2 + p_5 \omega^4 \end{aligned} \quad (7)$$

Polynomial $p(s)$ is Hurwitz stable if and only if

- The roots of even and odd parts are all real and interlacing
- The even and odd terms have the same sign of the highest coefficient terms

$$\begin{aligned} K_e^{\min}(\omega) &= X_0 - Y_2 \omega^2 + X_4 \omega^4 + X_8 \omega^8 - Y_{10} \omega^{10} + .. \\ K_e^{\max}(\omega) &= Y_0 - X_2 \omega^2 + Y_4 \omega^4 - X_6 \omega^6 + Y_8 \omega^8 - X_{10} \omega^{10} + .. \end{aligned} \quad (8)$$

$$\begin{aligned} K_o^{\min}(\omega) &= X_1 - Y_3 \omega^2 + X_5 \omega^4 - Y_7 \omega^6 + X_9 \omega^8 - Y_{11} \omega^{10} .. \\ K_o^{\max}(\omega) &= Y_1 - X_3 \omega^2 + Y_5 \omega^4 - X_7 \omega^6 + Y_9 \omega^8 - X_{11} \omega^{10} + .. \end{aligned} \quad (9)$$

$$K_e^{\min}(s) = X_0 + Y_2 s^2 + X_4 s^4 + Y_6 s^6 + X_8 s^8 + Y_{10} s^{10} + .. \quad (10)$$
$$K_e^{\max}(s) = Y_0 s + X_2 s^2 + Y_4 s^4 + X_6 s^6 + Y_8 s^{89} + X_{10} s^{10} + ..$$

$$K_o^{\min}(s) = X_1 s - Y_3 s^3 + X_5 s^5 + Y_7 s^7 + X_9 s^9 + Y_{11} s^{11} + .. \quad (11)$$
$$K_o^{\max}(s) = Y_1 s - X_3 s^3 + Y_5 s^5 + X_7 s^7 + Y_9 s^9 + X_{11} s^{11} + ..$$

$$K_1(s) = K_e^{\min}(s) + K_o^{\min}(s) = X_0 + X_1 s + Y_2 s^2 + Y_3 s^3 + .. \quad (12)$$

$$K_2(s) = K_e^{\min}(s) + K_o^{\max}(s) = X_0 + Y_1 s + Y_2 s^2 + X_3 s^3 + ..$$
$$K_3(s) = K_e^{\max}(s) + K_o^{\min}(s) = Y_0 + X_1 s + X_2 s^2 + Y_3 s^3 + .. \quad (13)$$
$$K_4(s) = K_e^{\max}(s) + K_o^{\max}(s) = Y_0 + Y_1 s + X_2 s^2 + X_3 s^3 + ..$$

If origin is excluded from the rectangle then thesystem is considered to be stable. If the origin is included then the system becomesunstable. This principle is called Zero Exclusion Principle [9].

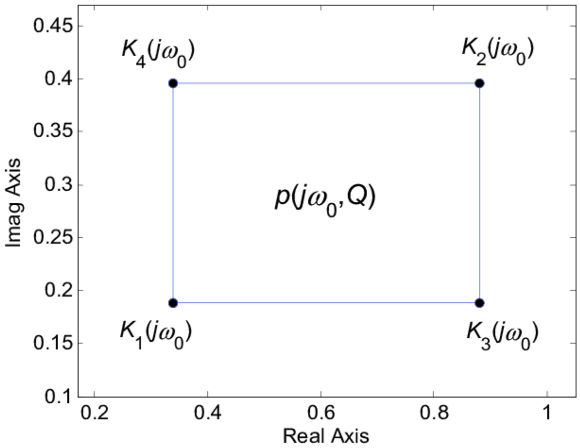

Figure 1: Kharitonov Rectangle

Generalized Kharitonov Theorem

Kharitonov theorem even though proves that the system is stable; sometimes the system may not be stable. In attempting to apply Kharitonov's theorem directly to control systems we encounter a certain degree of conservatism. This is mainly due to the fact that the characteristic polynomial coefficient perturb independently, whereas a crucial assumptions in Kharitonov's theorem is that coefficients of the characteristic polynomial vary independently.

Mikhailov's Criterion

A continuous-time polynomial

$p(s) = p_0 + p_1 s + .. + p_n s^n$ with $p_n > 0$ is stable if its frequency plot $p(j\omega)$

1. Starts on the positive real axis
2. Encircles the origin in a counter clockwise direction with a phase increment of $\frac{n\pi}{2}$ as

   $\omega \in [0, \infty]$

Numerical Example and Simulation

Let us consider a transfer function represented as $G(s) = \frac{k}{s(s+a)(s+b)}$ where

$k \in [220,180]$

$a \in [4.5,3.5]$

$b \in [7,5]$

The characteristics equation of the transfer function can be written as

$p(s) = k + abs + (a+b)s^2 + s^3$

Which is in the form $p(s) = A_0 + A_1 s + A_2 s^2 + A_3 s^3$

The boundary conditions of the parameters are
$$A_0 \in [180, 220]$$
$$A_1 \in [17.5, 31.5]$$
$$A_2 \in [8.5, 11.5]$$
$$A_3 = 1$$

The Kharitonov polynomials are

$$\begin{aligned} K_1(s) &= 180 + 17.5s + 11.5s^2 + s^3 \\ K_2(s) &= 180 + 31.5s + 11.5s^2 + s^3 \\ K_3(s) &= 220 + 17.5s + 8.5s^2 + s^3 \\ K_4(s) &= 220 + 31.5s + 8.5s^2 + s^3 \end{aligned} \quad (14)$$

Figure 2 represents the kharitonov rectangles for the above example.

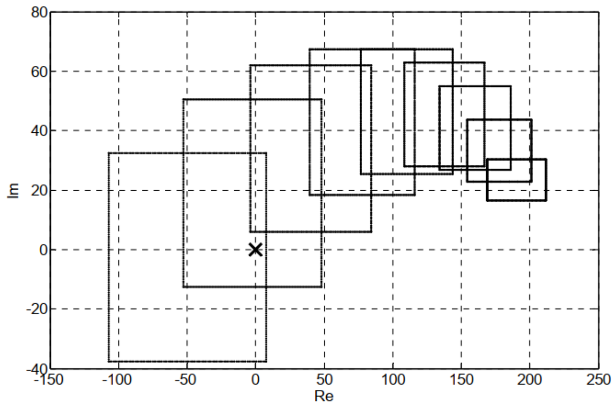

Figure 2: Kharitonov rectangle for Example-1

Consider a 2nd numeric example where the characteristics equation is

$$p(s,q)=[0.25,1.25]+[0.75,1.25]s+[2.75,3.25]s^2+[0.25,1.25]s^3 \quad (15)$$

By solving the said, equations, we get the kharitonov rectangle for 2nd numeric example

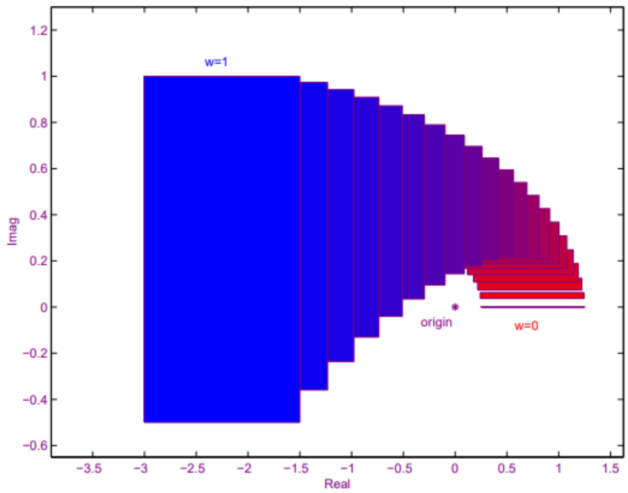

Figure 3: Kharitonov rectangle for 0<w<1

Conclusion

This paper provides an analysis of classical Kharitonov theorem and its proof along with generalized Kharitonov theorem. Proof of all the theorem has been provided. Simulation results have been provided to validate the proof.

Acknowledgement

The authors are thankful to the research mentors of "Accendere Knowledge Management Services Pvt Ltd", CL Educate Limited for their help in this paper.

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