Magnetohydrodynamics unsteady two phase flow between two parallel walls

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Abstract:

Various situations involving to petroleum industry, aeronautics, plasma physics and geophysics and in industrial applications all that absorb multi fluid flow situations. For instance, in geophysics one may be interested in studying the geomagnetic field with the hot springs (fluids) in geothermal regions. And, once he knows the interaction of the geomagnetic field with the flow field, he knows how to easily obtain the temperature division as of the so called energy equation. The problem of an unsteady MHD two immiscible fluids flow and heat transfer throughout a horizontal channel is investigated analytically in the presence of an applied magnetic field transverse to the flow direction. It is assumed that the fluids in the two regions are incompressible, immiscible and electrically conducting, having different viscosities, electrical conductivities. With these assumptions and considering that the magnetic Reynolds number is small the basic equations of motion, current, the no-slip boundary conditions at the walls and interface conditions between the two-fluid regions have been formulated. The resulting governing linear differential equations are solved analytically, using the prescribed boundary and interface conditions to get the exact solutions for velocity distributions such as primary and secondary distributions in both regions. Also, their corresponding numerical results for various set of standards of the governing parameters are obtain to represent them graphically and are discussed in detail.

Mathematical formulation of the governing equations of motion, Energy, boundary and interface conditions:

By choosing an unsteady hydromagnetic two fluid flow in a horizontal channel containing of two infinite parallel walls expand by the wall of the x- and z-directions known by y = h_1 and y = $-h_2$, the physical pattern and flow model choosing the origin is the halfway involving the walls. The regions $0 \le y \le h_1$ and $-h_2 \le y \le 0$ are in use by two immiscible electrically conducting, incompressible fluids having different viscosities μ_1, μ_2, and electrical conductivities σ_1, σ_2. A constant magnetic field of strength B_0 is applied transverse to the flow direction i.e., along the y-direction. There is also, a constant electric field E_0 in the z-direction. The flow in both higher and lower regions is motivated by a regular constant pressure gradient $\left(-\frac{\partial p}{\partial x}\right)$. Further, it is assumed that the induced magnetic field is small when compared with the applied field, so that is negligible. Among these assumption, the non dimensional principal equations of motion current and energy and the corresponding boundary and crossing point situations for both phases are expressed (in a non –rotating frame of reference) as

Region-I:

$$\rho_1 \frac{\partial u_1}{\partial t} - \mu_1 \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial p}{\partial x} + \sigma_1 u_1 B_0^2 + \sigma_1 E_0 B_0 = 0$$

Region II:

$$\rho_2 \frac{\partial u_2}{\partial t} - \mu_2 \frac{\partial^2 u_2}{\partial y^2} + \frac{\partial p}{\partial x} + \sigma_2 u_2 B_0^2 + \sigma_2 B_0 E_0 = 0$$

At this time subscripts 1 and 2 correspond to the principles for Region-I and Region-II correspondingly, anywhere u_1, u_2 are the x-component of fluid velocity; in the two regions respectively, and the time is t. The border circumstances on velocity are the no slip boundary condition at the lower wall and an oscillatory one at the upper wall. The boundary conditions on temperature are isothermal conditions.

Hydrodynamic boundary and border conditions for the two fluids can then be written as

$$u_1(h_1) = 0, \qquad \text{for } t \le 0$$

$$= \text{Real}\,(\varepsilon e^{i\omega t}), \quad \text{for } t > 0.$$

$$u_2(h_2) = 0,$$

$$u_1(0) = u_2(0),$$

$$\mu_1 \frac{du_1}{dy} = \mu_2 \frac{du_2}{dy} \text{ at } y = 0,$$

Where ε (amplitude) is a small constant quantity such that $\varepsilon << 1$ and ω is the frequency of oscillation at the wall, and the perturbed fields initially are zero, since the system is at rest for $t \le 0$. the above equations have been non-dimensionalised using the following dimensionless quantities:

$$u^{\bullet}{}_1 = \frac{u_1}{u_p}, u^{\bullet}{}_2 = \frac{u_2}{u_p}, \quad y^{\bullet}{}_i = \left(\frac{y_i}{h_i}\right)(i = 1,2), \quad u_p = \left(\frac{\partial p}{\partial x}\right)\frac{h_1^2}{\mu_1},$$

$$H_a^2 \text{ (Hartmann number)} = B_0^2 h_1^2 \left(\frac{\sigma_1}{\mu_1}\right), \ t^* = \frac{\upsilon_1 t}{h_1^2}, \ w^* = \frac{w h_1^2}{\upsilon_1},$$

α (Ratio of the viscosities) = $\frac{\mu_1}{\mu_2}$, h (ratio of the heights)= $\frac{h_2}{h_1}$,

σ (Ratio of the electrical conductivities) = $\frac{\sigma_1}{\sigma_2}$,

R_e (Electric load parameter) = $E_0 / B_0 u_p$

With the above transformations and for simplicity neglecting the asterisks, the non-dimensional form of equations becomes:

Region-I:

$$\frac{du_1}{dt} - \frac{d^2 u_1}{dy^2} + H_a^2 \left(R_e + u_1 \right) - 1 = 0$$

Region-II:

$$\frac{du_2}{dt} - \frac{d^2 u_2}{dy^2} + H_a^2 h^2 \alpha\sigma \left(u_2 + R_e \right) - \alpha h^2 = 0$$

The non-dimensional forms of the velocity and interface boundary conditions become

$$u_1(+1) = 0, \qquad \text{for } t \le 0,$$

$$= \mathrm{Re}\left(\varepsilon e^{i\omega t}\right), \qquad \text{for } t > 0. \qquad (1)$$

$$u_2(-1) = 0 \qquad (2)$$

$$u_1(0) = u_2(0) \qquad (3)$$

$$\frac{du_1}{dy} = (1/\alpha h)\frac{du_2}{dy} \quad \text{at} \quad y = 0 \qquad (4)$$

Condition (2) represents the no-slip condition at the lower wall and the condition (1) is due to oscillation of the upper wall. Conditions (3) and (4) represent the continuity of velocity and shear stress at the interface y = 0.

Solutions of the problem

The governing momentum equation along with the energy equation and solved subject to the boundary and interface conditions for the velocity distributions in both regions. These equations are coupled partial differential equations that cannot be solved in a closed form. However, they can be reduced to ordinary differential equations by assuming the following two term series:

$$u_1(y,t) = u_{01}(y) + (\varepsilon \cos wt) u_{11}(y),$$
$$u_2(y,t) = u_{02}(y) + (\varepsilon \cos wt) u_{12}(y),$$

Results and discussion

The problem of an unsteady MHD two immiscible fluids flow and heat transfer through a horizontal channel is investigated analytically in the presence of an applied magnetic field transverse to the flow direction. The two fluids are assumed to be incompressible and electrically conducting possessing different viscosities, thermal and electrical conductivities. The resulting partial differential equations are reduced to ordinary linear differential equations and solved analytically by means of the assumed solutions to obtain exact solutions for the velocity distributions, such as, u_1, u_2 respectively in the two regions. The graphs for the velocity distributions for both steady state and unsteady state flow motions are shown in figures to discuss the important features of hydromagnetic state of the fluid. The solid lines show the profiles for an unsteady motion and the dash-dot lines the steady flow motion respectively.

The effect of varying the Hartmann number H_a on velocity distributions are exhibited in figs.1. From fig.1, it can be seen that the effect of increasing H_a is to increase the velocity distributions u_1 and u_2 in the two regions. That is, the velocity of fluid increases as the strength of the magnetic field increase, which implies that the body force is an accelerating force. Owing to this accelerating force the fluids temperature are actually increased and hence, at commencement of motion from this tendency is significant. The maximum velocity in the channel tends to move above the channel centre line towards region-I (i.e., upper fluid region) as H_a increases, when all the remaining governing parameters are fixed.

The effect of varying the height ratio h on velocity distributions is shown in fig. 2. It is found from fig. 2 that, an increase in the value of h increases the velocities u_1, u_2 in the two regions.

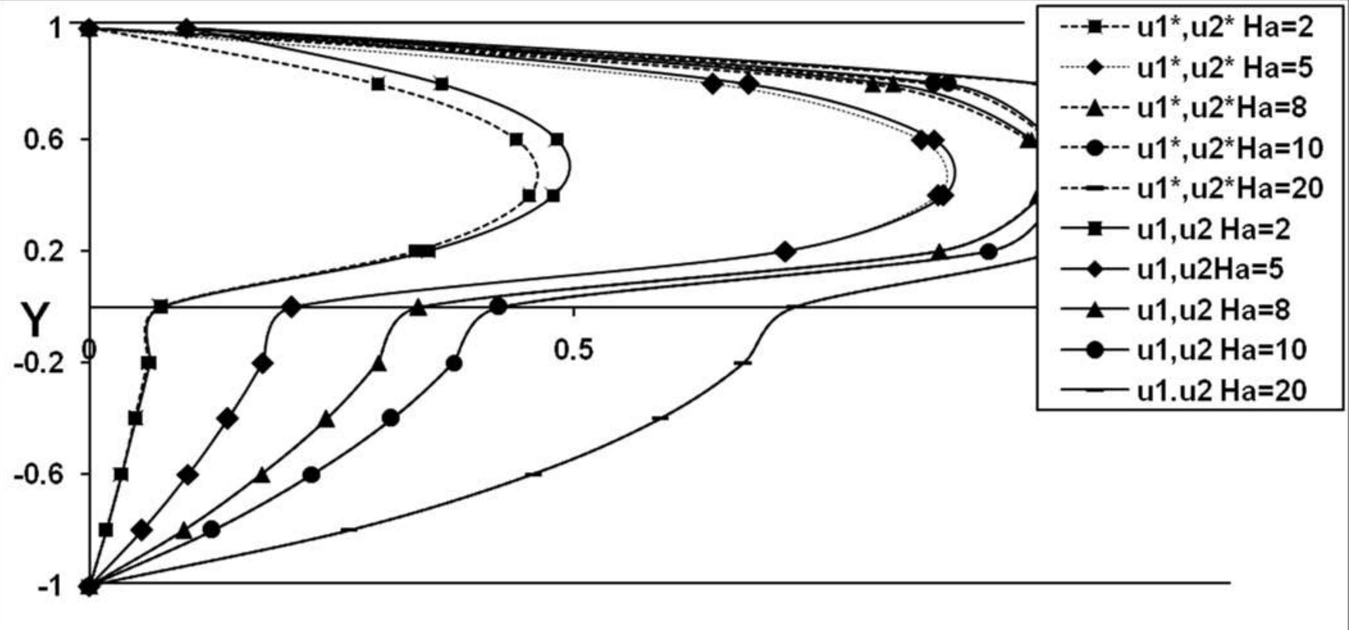

Fig.1 Velocity profiles u_1, u_2(unsteady flow),u_1^*, u_2^*(steady flow) for different H_a and α=0.333,σ=2,R_e=-1, h=0.1.

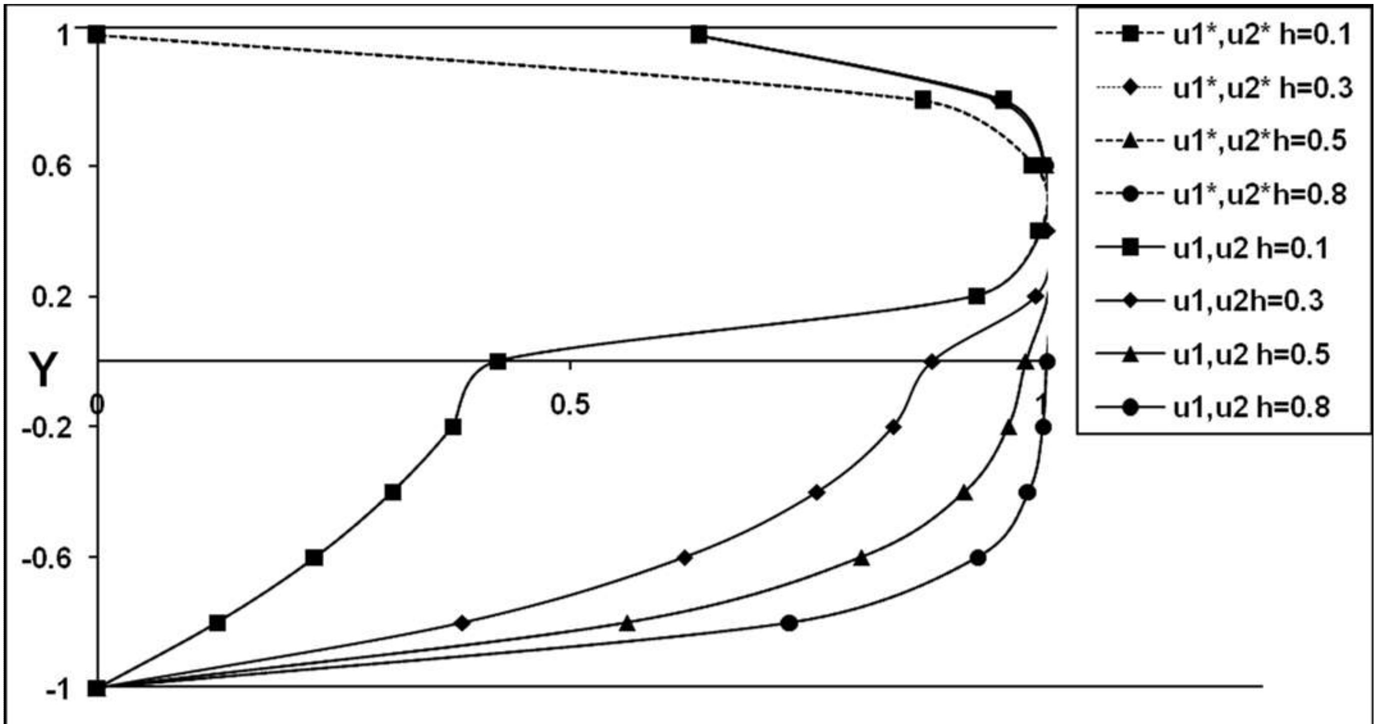

Fig.2 Velocity profiles u_1, u_2(unsteady flow),u_1^*, u_2^*(steady flow) for different h and H_a=10, α=0.333,σ=2,R_e=-1, h=0.1.

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